Read Me First!

1 Hints for doing labs

This section is to introduce you to the methods of experimental physics, as well as to familiarize you with techniques that are common to many of the labs.

The overall outline of a physics lab is:

1. Theoretical description of the phenomenon to be studied.
2. Experimental description of the how the phenomenon can be measured.
3. Specific information about the equipment used for measurement.
4. Procedures for making and recording measurements.
5. Analysis of the data to give experimental results.
6. Conclusions and discussion of results.

In the labs that follow, there are 'Pre-lab' exercises and questions to get you familiar with the theoretical and experimental basis for the labs; the pre-lab should be done **prior** to coming to lab, but not so far ahead of time that you forget what the lab is about by the time you actually do the lab.

While setting up and doing the lab, you should be asking yourself 'am I getting consistent results'? 'Where are the errors'? Dashing off some quick calculations and rough graphs while you are doing the lab can be very useful to let you know when things aren't going quite right. If you discover problems while you are doing the lab, you have a chance of correcting the problems, remeasuring, etc. A problem found later will be much harder to correct.

This is also why you should record as much information as you can about the conditions under which you are doing the lab, the setup, all measurements, etc. Sometimes the extra data (even if the procedure doesn't ask for it) provides the vital clue to let you untangle what *really* happened. Don't rely on your memory for this: *write it down!*

Doing your analysis and conclusions will be much easier if you've done some rough analysis during the lab, have written down everything, and given some thought to sources of error while setting up and performing the lab.

2 Measurements and Errors

Whenever you make a measurement of a quantity, there is inevitably some uncertainty or 'error' in the measurement.

As an example, suppose that you weigh yourself on a bathroom scale. What you measure is limited by your ability to accurately estimate fractions between the marks on the dial of the scale (if an analog scale) or in the number of digits displayed (if a digital scale). Perhaps the dial is moving slightly (or digits flickering) as you breath and move slightly, so taking a single measurement will have some uncertainty about how well it represents an 'average' weight.

In addition, one should ask how well the scale is calibrated: is a precisely 1 kg mass recorded as exactly 1.00 kg? Or does the scale show (for example) 1.03 kg?

All of these (and more) contribute **experimental error** to your measurement. One of your goals in these labs is to reduce the amount of experimental error to a minimum, and another is to quantify the amount of error that inevitably remains. While all measurements are subject to error, with a good understanding of the sources and amounts of the error, one can still get results that are a very good estimate of the underlying 'true' values.

3 Rounding and Significant Figures

If a friend tells you that they got on the scale and have a mass of 76 kg, one would be justified in thinking that the friend's 'true' mass is somewhere between 75.5 kg and 76.5 kg; because one would typically not weigh oneself with tremendous accuracy, and just round off to the nearest kilogram.

But if the friend tells you that they have a mass of 76.1254 kg, one would conclude that they *did* take extreme steps to measure their mass accurately. Similarly, if they say that they mass 'about 70 kg' one might conclude that it is only a rough estimate of their mass, based both on the 'about', and the fact that the mass is given to the nearest 10 kg.

In the cases above, one draws conclusions (about the friend's mass, or in the case of the extremely accurate mass, about their sanity) based on the number of **significant figures** given.

Significant figures are both a way of roughly indicating the error in a number, and a simple way of keeping track of how errors in measurements affect the results of calculations using those measurements.

When you report a value of a measurement, or of a calculation using a measurement, you should round off to the correct number of significant figures. To do otherwise is to misrepresent the accuracy of your measurements, and is a matter of scientific integrity rather than just a mathematical shortcut.

But how many significant figures is the right number? You have to round off the least significant (rightmost) digits, until the rightmost digit is about the accuracy of your measurement.

As an example, suppose that your friend really measured their mass with an accuracy of 100 g (0.1 kg). Then 76.1254 kg is too many significant figures (the accuracy of 0.1 kg is worse than 0.0001 kg), and one should round off to 76.1 kg. A digit is 'significant' when one's measurements are good enough to detect a change of that digit.

When reporting a number, the rightmost digits to the right of the decimal point are assumed to be significant, even if zero: if you report 76.100 kg, it implies a 0.001 kg measurement accuracy. Zeros to the left of the decimal point are assumed to be significant unless the decimal point is omitted. Thus 700. kg should be taken to mean '700 kilograms with accuracy of 1 kilogram', while 700 kg (note the missing '.') is '700 kilograms with accuracy 100 kilograms'. That little decimal point makes a big difference in the assumed accuracy!

The number of 'significant figures' is simply the number of digits that are 'significant'. For example: 76.1000 has 6 significant figures, 76.1 has three, 70. has two (notice the decimal point) and 70 has one.

How about a number like '0.00761'? Does it have 5 significant figures, or only 3? When counting significant figures, one should look at a number as if it is in *scientific notation*: a number between 1 and 10 multiplied by a power of ten. So 0.00761 in scientific notation is 7.61×10^{-3}, and it clearly has three significant figures. You don't have to use scientific notation everywhere, but it does help you keep the significant figures correct, and when numbers are very large or small scientific notation helps prevent you from accidentally dropping or adding a zero.

When doing calculations, one should round off results to the **smallest** number of significant figures of the data that went into the calculation. If you measure Joe's mass as 76.12 kg (4 sig.fig.) and his acceleration as $2.3\,\mathrm{m/s^2}$ (2 sig.fig.), the force ($F = ma$) on your calculator would give $175.076\,\mathrm{kg\,m/s^2}$ (6 sig.fig.), but the calculation is really only good to the minimum (2 sig.fig.) of the data you put into the calculation. In this case, you should report a force of $180\,\mathrm{kg\,m/s^2}$ with two significant figures.

One can also explicitly quote the errors in a measurement or calculation: 'Joe's mass was measured as 76.12 ± 0.03 kg', but the significant figures of both the measurement and the quoted error should be consistent. In other words, saying that the error is 0.03 kg doesn't let you quote the measurement as 76.1252345 kg, and you shouldn't say the error is 0.0321354 kg unless you've really determined it that accurately.

4 Accuracy and Statistical Error

Accuracy refers to the repeatability of a measurement. If you get on a scale 5 times in succession, how different are the measurements of your weight? Perhaps the scale reads 75.3, 75.6, 74.9, 75.0, 75.4 kilograms for the five measurements; the range of values (74.0 kg to 75.6 kg) gives you an indication of the accuracy of the measurements.

When you measure the same thing several times, you can improve accuracy by taking the average (or **mean**) of the N measurements:

$$\langle x \rangle = \frac{1}{N} \sum_{i=1}^{N} x_i \tag{1}$$

where the '$\langle x \rangle$' indicates 'the mean x', and x_i are the individual measurements of x.

For the weighing example above, the mean mass is 75.24 kg. But to quantify the accuracy, one has to calculate the **standard deviation** σ_x of the measurements of x:

$$\sigma_x^2 = \frac{1}{N-1} \sum_{i=1}^{N} (x_i - \langle x \rangle)^2 \tag{2}$$

by taking (squared) differences of the individual measurements and their overall mean.

Many scientific calculators have built-in functions that will calculate the mean and standard deviation for you; if so, you can just put in your data and let the calculator do the work.

The standard deviation calculated for the data above is 0.288 kg, which says that the accuracy of each of the individual measurements is about 0.3 kg. The accuracy of the mean is simply the standard deviation of individual measurements divided by the square root of the number of measurements:

$$\sigma_{\langle x \rangle} = \frac{\sigma_x}{\sqrt{N}}, \tag{3}$$

So the mean of our five weight measurements should be reported as 75.2 ± 0.1 kg, rounding off to a consistent number of significant figures.

This kind of 'random variations in measurements' is known as a 'statistical error', and can be reduced by taking the average of repeated measurements.

4.1 Precision, Systematic Error, and Bias

The **precision** of your measurements is how closely they represent the 'true' value you are trying to measure. This is not the same as accuracy: one can have very accurate (reproducible) measurements that are not very precise (they all differ from the true value), and precise measurements that have poor accuracy (the average is precise, but there is a lot of 'scatter' in the measurements).

To use our weighing example again, suppose that the scale reads low by 5 kg. We could have highly reproducible measurements, but there will always be a 5 kg error.

The 5 kg offset in the scale is an example of a **systematic error**: all of your measurements are 'systematically' low by 5 kg. We could remove this systematic error by **calibrating** the scale: putting some 'known' masses on the scale and recording their weight reading. This would give us the correspondence between 'what the scale reads' and 'what mass is on the scale'.

(There would still be some systematic error from how well known the 'known' masses are, but it would presumably be a much smaller error.)

Similarly one could have a small systematic error from tidal effects: the gravitational attraction of the moon would decrease the net weight measured. A theoretical calculation could eliminate this systematic error quite accurately, but it's unlikely to be a very large effect.

One particular form of systematic error is called **bias**. Bias comes from a systematic error that has the effect of pushing one's measurements consistently too high or low on average.

Again with our weighing example, suppose that you weigh yourself at different times of day, but you forget to empty your pockets before stepping on the scale. Sometimes your pockets are mostly empty, other times you have some coins in your pockets that increases your measured weight. The variation in how much you have in your pockets will decrease the reproducibility of the measurements (statistical error), but taking the mean of many measurements can overcome that. However, there will still be an overall bias towards larger weights (non-empty pockets always make you heavier, never lighter) that even many repeated measurements will not overcome. This is a case where careful experimental procedure can reduce or eliminate a source of error.

Systematic errors are generally more difficult to eliminate than statistical errors; the first step is to try and identify sources of systematic errors, estimate (through measurement or theory) the size of their effect, then try to remove the errors through improved procedures, calibrations, or additional measurements.

You will always have many possible sources of systematic errors, but the overall error is nearly always dominated by a single systematic error that is largest in magnitude. So one's effort in reducing systematic error should always be directed towards the largest error first: you'll never see the systematic error from tides when weighing someone unless you first get them to empty their pockets before stepping on the scale.